SOFT GLUON RESUMMATION
IN HEAVY FLAVOUR PRODUCTION[1]

Paolo Nason[2]

CERN, TH Division, Geneva, Switzerland

Abstract

I discuss the effect of resummation of soft gluons in hadronic production of high mass systems, and in particular in heavy flavour production. I show that in widely used x-space resummation formulae, spurious terms that grow factorially in the order of the perturbative expansion are present. These terms spoil the convergence of the perturbative expansion. I also show that it is possible to perform the soft gluon resummation in such a way that these terms are not present. Implications for top and high mass dijet production at the Tevatron are discussed.

---

[1]Talk presented at the 10$^{\mathrm{th}}$ Rencontres de Physique de la Vallée d'Aoste, La Thuile, Val d'Aosta, March 3-9, 1996.

[2]On leave of absence from INFN, Sez. di Milano, Milan, Italy

1   Introduction

In this talk I will deal with the problem of the resummation of logarithmically enhanced effects in the vicinity of the threshold region in hard hadroproduction processes. Drell–Yan lepton pair production has been in the past the best studied example of this sort [1, 2, 3]. The threshold region is reached when the invariant mass of the lepton pair approaches the total available energy. A large amount of theoretical and phenomenological work has been done on this subject. References [4] and [5] summarize all the theoretical progress performed in this field. Resummation formulae have also been used in estimating heavy flavour production [6], [7]. In this case only a leading logarithmic resummation formula is known. Calculations of the next-to-leading logarithms are in progress [8].

Here I will not deal with sophisticated higher order effects calculations. I will instead describe some recent progress [9, 10] in the understanding of how to implement the resummation.

2   What are soft gluon effects

Coloured particles emit soft gluons with high probability. Normally the effect of soft gluon emission is small (at least in inclusive quantities), since they only slightly affect the kinematic of a process. However, in a production process of high mass objects, when we approach the threshold, soft gluon emission becomes important. Let us fix our attention on heavy flavour production in hadronic collisions near threshold. The process is schematically depicted in fig. 1. The incoming protons make a big effort in
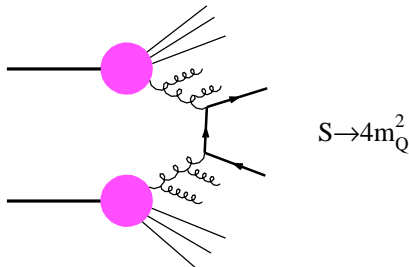

Figure 1: Heavy flavour production near threshold

providing partons with a large fraction of the longitudinal momentum, thus going towards the very large x region of the structure functions. Under these circumstances, even the

small amount of energy wasted by soft gluon radiation yields an important suppression of the cross section. In the usual application of the factorization theorem, part of the radiative corrections due to gluon radiation are included in the structure functions. Thus, depending upon the factorization scheme, the left-over suppression may be positive or negative. In the $\overline{\text{MS}}$ and DIS scheme the left-over is negative, so that the suppression effect appears instead as an enhancement of the cross section.

Let us now focus upon the case of heavy flavour production. The perturbative expansion for the partonic cross section at order $\mathcal{O}(\alpha_s^3)$, neglecting obvious indices (incoming parton types, etc.), has the structure

$$\hat{\sigma}(\hat{s}) = \sigma_0(\hat{s}) \left[1 + C\alpha_s \log^2(1 - \hat{\rho}) + \mathcal{O}(\alpha_s \log(1 - \hat{\rho}), \alpha_s^2) \right] , \qquad (1)$$

where $\hat{s}$ is the square of the partonic centre-of-mass energy, $\hat{\rho} = 4m^2/\hat{s}$, and $\sigma_0(\hat{s})$ is the Born cross section

$$\sigma_0(\hat{s}) = \frac{\alpha_s^2}{m^2} f^{(0)}(\hat{\rho}) . \qquad (2)$$

The scale at which α_s is evaluated is $\alpha_s = \alpha_s(m^2)$, unless the argument is explicitly given. The reader can find in ref. [11] explicit formulae for the functions $f^{(0)}(\hat{\rho})$, as well as for the constant coefficient C. The precise value of C depends upon the type of incoming partons (i.e. quarks or gluons) and upon the factorization scheme. In both the $\overline{\text{MS}}$ and the DIS scheme it is positive, so that in the following we can think of it as being a positive constant. Resummation, according to ref. [6], gives

$$\hat{\sigma}^{(\text{res})}(\hat{s}) = \frac{\alpha_s^2}{m^2} f^{(0)}(\hat{\rho}) \exp\left[C\alpha_s(s') \log^2(1 - \hat{\rho}) \right] \qquad (3)$$

where s' is a scheme-dependent function of $\hat{s}$ that goes to zero as $\hat{s} \to 4m^2$. We have $s' = (1 - \hat{\rho})^\eta m^2$, where $\eta = 1$ in the DIS scheme, and $3/2$ in the $\overline{\text{MS}}$ scheme. Formula (3) is supposed to include all terms of order $\alpha_s^m \log^n(1 - \hat{\rho})$ with $n > m$. Remember in fact that

$$\alpha_s(s') = \frac{1}{b_0 \log \frac{s'}{\Lambda^2}} = \frac{1}{b_0 \log \frac{m^2}{\Lambda^2} + b_0 \eta \log(1 - \hat{\rho})} = \alpha_s \left(1 - \alpha_s b_0 \eta \log(1 - \hat{\rho}) + \ldots \right) . \qquad (4)$$

so that in the exponent there are terms with arbitrary powers of α_s, and a power of the logarithm which is always larger than the power of α_s. The first subleading terms have the form $\alpha_s^k \log^k(1 - \hat{\rho})$.

In order to get a physical cross section the partonic cross section given above should be convoluted with parton luminosities

$$\sigma^{(\text{res})} = \int \mathcal{L}(\tau) \, \hat{\sigma}^{(\text{res})}(\tau S) \, d\tau = \frac{\alpha_s^2}{m^2} \int \mathcal{L}(\tau) f^{(0)}(\hat{\rho}) \exp\left[C\alpha_s \left((1 - \hat{\rho})^\eta m^2 \right) \log^2(1 - \hat{\rho}) \right] d\tau \qquad (5)$$

2

where $\hat{\rho} = 4m^2/(\tau S)$, and (omitting obvious parton indices)

$$\mathcal{L}(\tau) = \int F(x_1)F(x_2)\delta(x_1 x_2 - \tau)\, dx_1 dx_2\,. \tag{6}$$

Here we can spot a problem in the resummation formula. When performing our integral over τ, as $\tau \to \rho$ the argument of α_s in the exponential approaches zero. Before it actually hits the zero, it will hit a singularity in the running coupling $\alpha_s(s')$, causing the integral to diverge. In order to avoid the divergence a cutoff μ_0 was introduced in the literature [6, 12]. Observe that this cutoff has nothing to do with the standard factorization and renormalization scale μ. It is essentially a cutoff on soft gluon radiation, imposed in order to avoid the blowing up of the running coupling associated with soft gluon emission.

The use of a cutoff seems an ad hoc procedure in this case. It can however be justified to some extent. Suppose, for example, that we end up in a QCD calculation with a formula like

$$G = \int_0^{Q^2} \alpha_s(k^2)G(k^2)dk \tag{7}$$

where $G(k^2)$ is a smooth function as $k^2 \to 0$. Integrals of this kind are often found, for example, in the computation of shape variables in jet physics. This expression also is divergent as $k^2 \to 0$, since at some point α_s approaches the Landau pole. The divergence can be handled by a cutoff μ_0, which has to be large enough for α_s to be barely perturbative. For example, we may choose $\mu_0 = 5\Lambda$, a value around 2 GeV. We can then argue that

$$G = \int_{\mu_0}^{Q^2} \alpha_s(k^2)\, G(k^2)\, dk + C\frac{\mu_0}{Q}\,. \tag{8}$$

In fact, the divergence of the 1-loop expression of α_s does not signal a real physical divergence. More likely, the point at which α_s becomes of order 1 signals the breakdown of perturbation theory. We therefore exclude this region, estimate its contribution by dimensional analysis, and obtain a power correction. A slightly more formal justification makes use of the concept of IR (infrared) renormalons. We expand eq. (7) in powers of $\alpha_s(Q^2)$, using

$$\alpha_s(k^2) = \frac{1}{b_0 \log \frac{k^2}{\Lambda^2}} = \frac{1}{b_0 \log \frac{Q^2}{\Lambda^2} + b_0 \log \frac{k^2}{Q^2}} = \alpha_s \sum_{j=0}^{\infty} \left(-\alpha_s b_0 \log \frac{k^2}{Q^2}\right)^j\,, \tag{9}$$

and we get

$$\begin{aligned} G &= \int_0^{Q^2} \alpha_s \sum_{j=0}^{\infty} \left(-\alpha_s b_0 \log \frac{k^2}{Q^2}\right)^j \\ &= \alpha_s \sum_{j=0}^{\infty} (\alpha_s b_0)^j \int_0^{\infty} t^j e^{-t/2} dt \end{aligned} \tag{10}$$

3

where $t = \log Q^2/k^2$. The integral can be performed, and one gets

$$G = 2\alpha_s \sum_{j=0}^{\infty} j! \, (2\alpha_s b_0)^j \ , \tag{11}$$

which is a divergent series, since a factorial grows faster than any power. This lack of convergence is in fact a general feature of the perturbative expansion in field theory. The perturbative expansion should be interpred as an asymptotic one. The terms of the expansion (11) decrease for moderate values of j. As j grows the factorial takes over, the terms stop decreasing and begin to increase. This happens at the value of j at which the next term is equal to the current one

$$(j+1)! \, (2\alpha_s b_0)^{j+1} = j! \, (2\alpha_s b_0)^j \tag{12}$$

or roughly

$$j_{\min} = \frac{1}{2\alpha_s b_0} \ . \tag{13}$$

Asymptotic expansions are usually handled by summing their terms as long as they decrease. Of course, in this resummation prescription there is an ambiguity, which is of the order of the size of the first neglected term. In our case

$$j_{\min}!(2\alpha_s b_0)^{j_{\min}} \approx e^{j\log j - j} \frac{1}{j^j} = e^{-\frac{1}{2\alpha_s b_0}} \approx \frac{\Lambda}{Q} \ , \tag{14}$$

which gives a power-suppressed ambiguity with the same power law that we found using the cutoff procedure.

From the discussion given above, we would expect that the resummation formulae should include a cutoff of the order of a typical hadronic scale, and varying the cutoff within a factor of order 1 should affect the cross section by terms of the order Λ/Q. In fact, this is not the case. The cutoff has a dramatic effect on the cross section, as can be seen from figs. 2 and 3 of ref. [6]. For example, the uncertainty band obtained by varying the scale μ_0 between $0.2m$ and $0.3m$ for top production in the gg channel brings about a change in the cross section of a factor of 2, for a top mass between 100 and 200 GeV. These two values of μ_0 correspond to cutting off the gluon radiation at energies of the order of 20 to 30 GeV, therefore much larger than a typical hadronic scale. Other proposals for the resummation procedure have appeared in the literature. In ref. [13] a method was developed in the context of Drell–Yan pair production, and it was applied to the heavy flavour case in ref. [7]. Also in ref. [7] (as can be seen from formula (116) and the subsequent discussion) unphysically large cutoffs are present, much larger than the typical hadronic scale that one expects.

In the following section, I will show that the presence of large cutoffs and of large ambiguities in the resummation formula, is not at all related to the blowing up of the coupling constant. In other words, there are other sources of factorial growth of the perturbative expansion for the resummation of soft gluons, and they largely dominate the factorial growth due to the running coupling. I will also show that these terms are spurious, and that soft gluon resummation can be easily formulated in such a way that these terms are not present.

3   Problems with the x space resummation formula

For definiteness, let us focus upon the resummation formula (3). We pointed out earlier that this formula is divergent when the argument of α_s becomes too small, and the coupling constant blows up. In fact, formula (3) is divergent even for fixed coupling constant. At fixed coupling it can be written as

$$
\begin{aligned}
\sigma^{\text{(res)}} &= \frac{\alpha_s^2}{m^2} \int d\tau \, \mathcal{L}(\tau) f^{(0)}(\hat{\rho}) \exp\left[\alpha_s C \log^2(1-\hat{\rho})\right] \\
&= \frac{\alpha_s^2}{m^2} \int d\hat{\rho}_\rho^1 \frac{\rho}{\hat{\rho}^2} \mathcal{L}\left(\rho/\hat{\rho}\right) f^{(0)}(\hat{\rho}) \exp\left[\alpha_s C \log^2(1-\hat{\rho})\right] ,
\end{aligned}
\tag{15}
$$

where $\rho = 4m^2/S$, and the integral diverges as $\hat{\rho} \to 1$, since the exponential

$$
\exp(a \log^2(1-\hat{\rho})) = (1-\hat{\rho})^{-a \log(1-\hat{\rho})}
\tag{16}
$$

grows faster than any inverse power of $1-\hat{\rho}$ as $\hat{\rho} \to 1$. This divergence can again be related to factorial growth in the perturbative expansion. Expanding formula (15) in powers of α_s we get

$$
\sigma^{\text{(res)}} = \frac{\alpha_s^2}{m^2} \sum_{k=0}^{\infty} \frac{1}{k!} (C\alpha_s)^k \int_\rho^1 d\hat{\rho} \, \frac{\rho}{\hat{\rho}^2} \mathcal{L}\left(\rho/\hat{\rho}\right) f^{(0)}(\hat{\rho}) \log^{2k}(1-\hat{\rho}) .
\tag{17}
$$

It is easy now to see that, due to its singularity for $\hat{\rho} \to 1$ the integral of $\log^{2k}(1-\hat{\rho})$ grows like $(2k)!$. Let us make here the simplifying assumption that $f^{(0)}(\hat{\rho}) \approx \theta(1-\hat{\rho})$. In a neighbour of the singularity, the integral has the form

$$
\int_{1-\epsilon}^1 d\hat{\rho} \, \log^{2k}(1-\hat{\rho}) = \int_{\log 1/\epsilon}^\infty dt \, e^{-t} t^{2k} .
\tag{18}
$$

where we have performed the substitution $t = \log 1/(1-\hat{\rho})$. The above integral equals

$$
\int_{\log 1/\epsilon}^\infty dt \, e^{-t} t^{2k} = \int_0^\infty dt \, e^{-t} t^{2k} - \int_0^{\log 1/\epsilon} dt e^{-t} t^{2k} = (2k)! \ - \int_0^{\log 1/\epsilon} dt \, e^{-t} t^{2k} .
\tag{19}
$$

Since

$$\int_0^{\log 1/\epsilon} dt \, e^{-t} t^{2k} < (\log 1/\epsilon)^{2k+1} \tag{20}$$

we see that the contribution to the integral near the singularity is dominated by the term $(2k)!$. The power expansion for the cross section is then

$$\sigma^{(\text{res})} \approx \sum_{k=0}^{\infty} \frac{(2k)!}{k!} \left(C\alpha_s\right)^k \approx \sum_{k=0}^{\infty} k! \left(4C\alpha_s\right)^k . \tag{21}$$

The above formula is in fact appropriate for the case of heavy flavour cross section with a lower cut on the invariant mass of the pair[3]. As in the previous example, the factorial growth of formula (21) will give rise to ambiguitites in the resummation of the perturbative expansion. These ambiguities are not, however, related to renormalons, since they occur also at fixed coupling constant. In fact, they are in general fractional powers of Λ/Q, where Q is the scale involved in the problem, and Λ is a typical hadronic scale. For example, in the case of heavy flavour production at fixed invariant mass of the heavy quark pair via the gluon-fusion mechanism, the ambiguity would have the form $(\Lambda/Q)^{0.16}$, and it would thus be extremely relevant even for very massive heavy flavour pairs.

These large terms in the perturbative expansion are in fact spurious. They are an artefact of the x space resummation procedure. This can be easily understood with the following argument. Exponentiation of the gluon emission is possible because, roughly speaking, each soft gluon is emitted independently. This independence is however only approximate: the total momentum must be conserved. Momentum conservation, however, is a subleading effect in the soft resummation formula. Yet, its violation leads to factorially growing terms. These terms are subleading from the point of view of the logarithmic behaviour, but very important from the point of view of the factorial growth of the perturbative expansion. The presence of large factorial terms due to momentum non-conservation can be understood also by simple arguments. The emission of k gluons, where each gluon has a limit on its energy $E_i < \eta$, leads to a phase space that is larger by a factor of $k!$ than the case when the total energy of emission is bounded $\sum E_i < \eta$. Thus phase space alone provides a $k!$ factor. We can see in more detail the origin of the $(2k)!$ term by considering the formula for the partonic cross section with two emitted soft gluons, implementing momentum conservation. We have

$$\sigma^{(2)}(\hat\rho) = \frac{1}{2}(2C\alpha_s)^2 \frac{\alpha_s^2}{m^2} \int \left[\frac{\log(1-z_1)}{1-z_1}\right]_+ \left[\frac{\log(1-z_2)}{1-z_2}\right]_+ f^{(0)}(\hat\rho') \, \delta(\hat\rho - z_1 z_2 \hat\rho') \, dz_1 dz_2 d\hat\rho' . \tag{22}$$

---

[3]A more accurate analysis, using the known behaviour of $f^{(0)}$ near threshold would have yielded an extra factor of $4/9$ in front of C in eq. (21).

The leading logarithmic term of the above integral is given by

$$\sigma^{(2)}(\hat\rho) = \frac{1}{2}(2C\alpha_s)^2\frac{\alpha_s^2}{m^2}\log^4(1-\hat\rho)\,f^{(0)}(\hat\rho) + \dots\,, \qquad (23)$$

where terms with less than 4 powers of logarithms are neglected. We now see that the integral of the leading logarithmic term of $\sigma^{(2)}(\hat\rho)$ in $\hat\rho$ has a large factor $\approx 4!$ due to the integral of $\log^4(1-\hat\rho)$, while the integral of the full expression, eq. (22), gives

$$\int_0^1 d\hat\rho\,\sigma^{(2)}(\hat\rho) = \frac{1}{2}(2C\alpha_s)^2\frac{\alpha_s^2}{m^2}\left(\int_0^1 dz\left[\frac{\log(1-z)}{1-z}\right]_+\right)^2\int_0^1 d\hat\rho'\,f^{(0)}(\hat\rho)\ = 0\,, \qquad (24)$$

due to the vanishing of the integrals of the soft emission factors with the + prescription. In general, we see that if we take generic moments of $\sigma^{(k)}(\hat\rho)$ (i.e. $\int\hat\rho^m d\hat\rho\,\sigma^{(k)}(\hat\rho)$), the leading log expression grows like $(2k)!$, while the full expression grows only geometrically with k.

The criticism described so far applies to the calculations of soft gluon effects in heavy flavour production given in refs. [6] and [7]. As one may expect, the large factorial terms give rise to large corrections to the cross section. Since the term of the perturbative expansion grow strongly with the order, they also give large uncertainties. In ref. [6], the presence of large uncertainties is in fact recognized. In ref. [7] it is claimed that the uncertainties are small. Even there, however, an unphysically large cutoff is needed in order to make sense out of the resummation formulae.

A second, more subtle criticism of the resummation formulae has to do with the presence of $1/Q$ corrections that arise from infrared renormalons. It was shown in ref. [14] that soft gluon resummation does not yield the correct renormalon structure of the Drell–Yan cross section. This proof was given in a simplified framework in which the renormalon structure can be computed exactly. This result suggests the absence of $1/Q$ corrections in Drell–Yan cross sections, an issue that is still much debated in the literature [15]–[18]. We would like to remark, however, that even if $1/Q$ corrections were present in Drell–Yan and heavy flavour production, they would only be of the order of 1% for top production at the Tevatron.

It is possible to formulate the resummation of soft gluons in such a way that the kinematic constraints are explicitly satisfied, and no factorial growth arises in the perturbative expansion. It is enough to formulate the resummation problem in the Mellin transform space[4].

---

[4]In fact, resummation formulae are usually derived in Mellin space.

In ref. [9] we specify a resummation prescription that uses the Mellin space formula. We demonstrate that with such prescription there are no factorially growing terms in the resummed perturbative expansion. Yet, this prescription consistently includes soft effects. We call this prescription "Minimal Prescription" (MP from now on), because it does not introduce large terms that are not justified by the soft gluon approximation.

4 Phenomenological applications

We have computed various heavy flavour production cross sections using our MP formula. Details are given in refs. [9, 10]. Here I report the most salient results.

The importance of the resummation effects is illustrated in figs. 2, 3 and 4, where we plot the quantities

$$\frac{\delta_{\mathrm{gg}}}{\sigma_{\mathrm{NLO}}^{(gg)}} , \quad \frac{\delta_{\mathrm{q\bar{q}}}}{\sigma_{\mathrm{NLO}}^{(q\bar{q})}} , \quad \frac{\delta_{\mathrm{gg}} + \delta_{\mathrm{q\bar{q}}}}{\sigma_{\mathrm{NLO}}^{(gg)} + \sigma_{\mathrm{NLO}}^{(q\bar{q})}} . \tag{25}$$

Here δ is equal to our MP-resummed hadronic cross section in which the terms of order α_s^2 and α_s^3 have been subtracted, and $\sigma_{(\mathrm{NLO})}$ is the full hadronic NLO cross section. The results for b at the Tevatron can be easily inferred from fig. 2, since the $q\bar{q}$ component is negligible at Tevatron energies.

For top production, we see that in most configurations of practical interest, the contribution of resummation is very small, being of the order of 1% at the Tevatron. A complete review of top quark production at the Tevatron, based upon these findings, has already been given in ref. [10]. We also observe that, for top production at the LHC, soft gluon resummation effects are negligible. Of course, in this last case, there are other corrections, not included here, that may need to be considered. Typically, since the values of x involved are small in this configuration, one may have to worry about the resummation of small-x logarithmic effects [19].

We see from the figures that in most experimental configurations of interest these effects are fully negligible. One noticeable exception is b production at HERAb, at $\sqrt{S} = 39.2$, where we find a 12% increase in the cross section. This correction is however well below the uncertainty due to higher order radiative effects. For example, from the NLO calculation with the MRSA′ [20] parton densities and $m_b = 4.75\,\mathrm{GeV}$, we get $\sigma_{b\bar{b}} = 10.45\,^{+8.24}_{-4.65}$ nb, a range obtained by varying the renormalization and factorization scales from $m_b/2$ to $2m_b$. Thus the upper band is 80% higher than the central value, to be compared with a 10% increase from the resummation effects. This result is much less dramatic than the results of ref. [21].
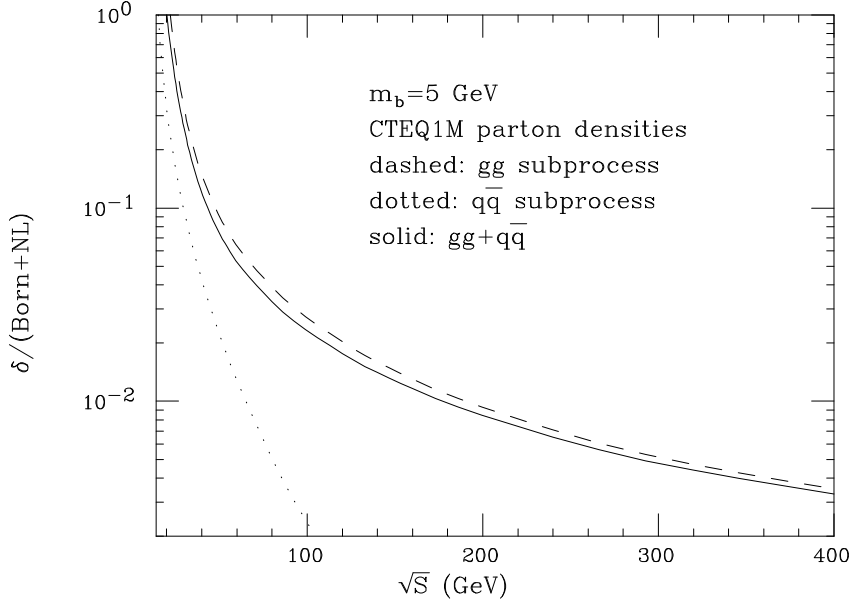
Figure 2: *Contribution of gluon resummation at order α_s^4 and higher, relative to the NLO result, for the individual channels and for the total, for bottom production as a function of the CM energy in pp collisions.*

5 Jet Cross Sections

The interest in the effects of resummation on the behaviour of jet cross sections at large energy is prompted by the discrepancy between the single-inclusive jet-p_T distribution at large p_T, as measured by CDF [22], and the result of the NLO QCD predictions [23]. For simplicity we will study the effects of soft gluon resummation on the invariant mass distribution of the jet pair, which is, from a theoretical point of view, very close to the Drell–Yan pair production. Observe that other distributions, such as the p_T of the jet, have a rather different structure from the point of view of soft gluon resummation. In fact, while the jet pair mass is only affected by the energy degradation due to initial state radiation, the p_T of the jet may also be affected by the transverse momentum generated by initial state radiation, and by the broadening of the jet due to final state radiation.

A study of the jet pair mass distribution is not of purely academic interest, since also for this variable an analogous discrepancy between data and theory has been observed [24]. Studies of resummation effects in the inclusive p_T spectrum of jets are in progress (M. Greco and P. Chiappetta, private communication).
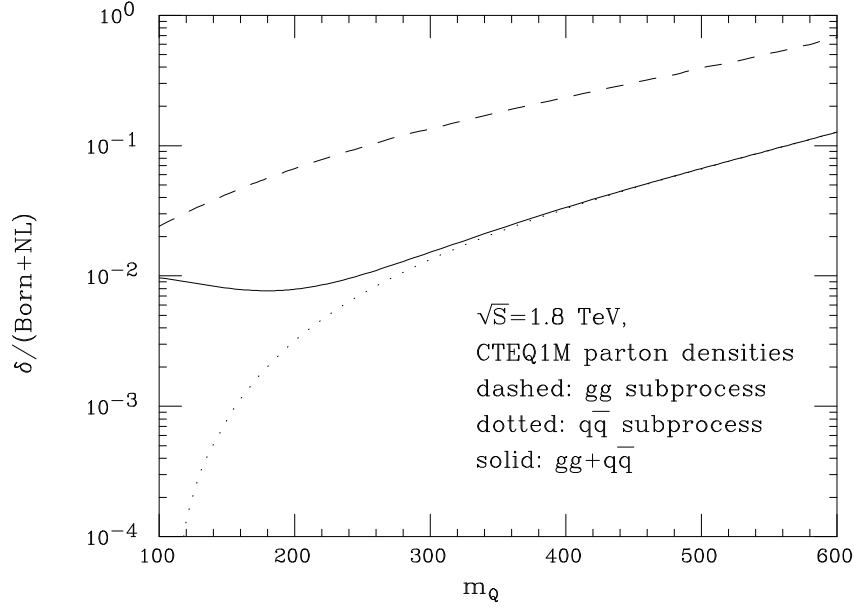
9

Figure 3: *Contribution of gluon resummation at order α_s^4 and higher, relative to the NLO result, for the individual subprocesses and for the total, as a function of the top mass in $p\bar{p}$ collisions at 1.8 TeV.*
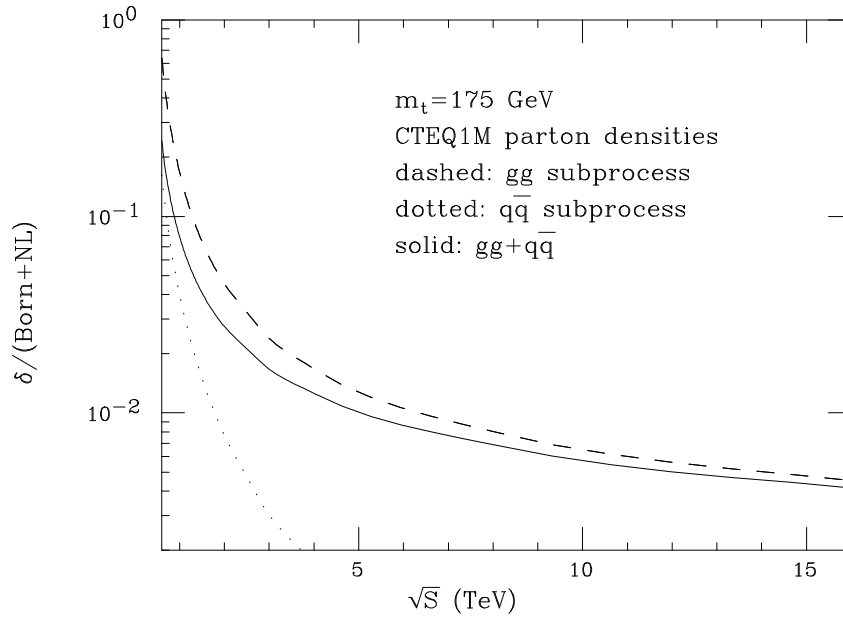

Figure 4: *Contribution of gluon resummation at order α_s^4 and higher, relative to the NLO result, for the individual channels and for the total, as a function of the CM energy in pp collisions.*

10

In fig. 5 we show the following quantities:

$$\frac{\delta_{\rm gg}^{(3)}}{\sigma^{(2)}}, \quad \frac{\delta_{\rm qg}^{(3)}}{\sigma^{(2)}}, \quad \frac{\delta_{\rm q\bar{q}}^{(3)}}{\sigma^{(2)}}, \quad \frac{\delta_{\rm gg}^{(3)} + \delta_{\rm qg}^{(3)} + \delta_{\rm q\bar{q}}^{(3)}}{\sigma^{(2)}} \tag{26}$$

where $\delta^{(3)}$ is equal to our MP resummed hadronic cross section in which the terms of order α_s^2 have been subtracted, and $\sigma^{(2)} = \sigma_{\rm gg}^{(2)} + \sigma_{\rm qg}^{(2)} + \sigma_{\rm q\bar{q}}^{(2)}$ is the full hadronic LO cross section (of order α_s^2). We use as a reference renormalization and factorization scale for our results $\mu = M_{jj}/2$. Notice that for large invariant masses the effects of higher orders are large. To understand how much is due to the first order corrections (which are exactly calculable [25]) and how much is due to corrections of order α_s^4 and higher, we show the following quantities in fig. 6

$$\frac{\delta_{\rm gg}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{\rm qg}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{\rm q\bar{q}}^{(4)}}{\sigma^{(3)}}, \quad \frac{\delta_{\rm gg}^{(4)} + \delta_{\rm qg}^{(4)} + \delta_{\rm q\bar{q}}^{(4)}}{\sigma^{(3)}}, \tag{27}$$

where $\delta^{(4)}$ is now equal to the MP resummed hadronic cross section with terms of order α_s^3 subtracted, and $\sigma^{(3)}$ is an approximation to the full NL cross section, summed over all subprocesses, obtained by truncating the resummation formula at order α_s^3. This figure shows that indeed most of the large K factor is due to the pure NLO corrections, with the resummation of higher order soft gluon effects contributing only an additional 10% at dijet masses of the order of 1 TeV.

These results should only be taken as an indication of the order of magnitude of the correction, since we have not included here a study of the resummation effects on the determination of the parton densities. From this preliminary study it seems however unlikely that the full 30–50% excess reported by CDF for jet p_T's in the range 300–450 GeV could be explained by resummation effects in the hard process. It is possible that the remaining excess be due to the poor knowledge of the gluon parton densities at large x, an idea pursued by the CTEQ group [26].
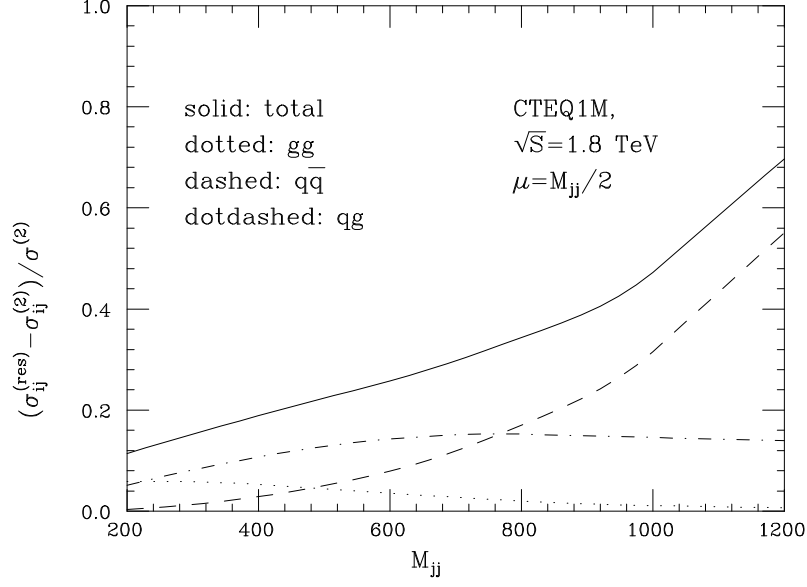
solid: total
dotted: gg
dashed: q$\bar{\text{q}}$
dotdashed: qg

CTEQ1M,
$\sqrt{S}$=1.8 TeV
μ=M$_{jj}$/2

$(\sigma_{ij}^{(res)} - \sigma_{ij}^{(2)})/\sigma^{(2)}$

M$_{jj}$

Figure 5: *Contribution of gluon resummation at order α_s^3 and higher, relative to the LO result, for the invariant mass distribution of jet pairs at the Tevatron.*
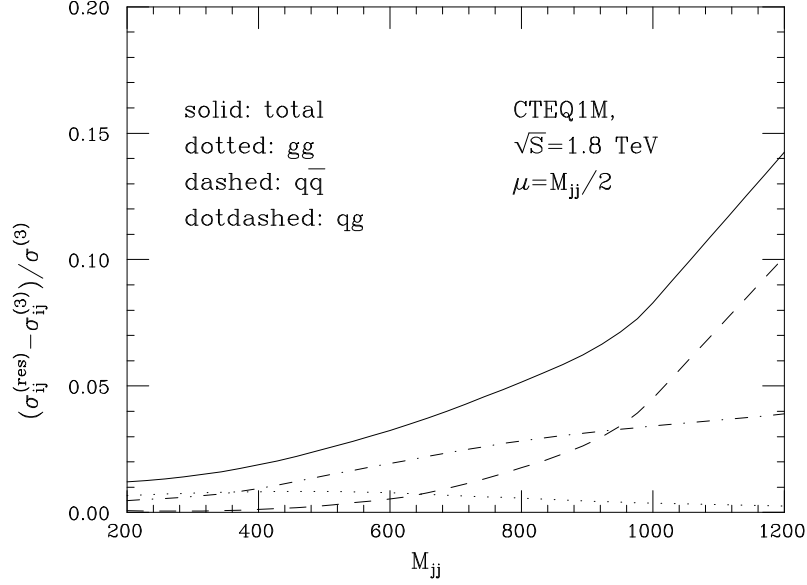
solid: total
dotted: gg
dashed: q$\bar{\text{q}}$
dotdashed: qg

CTEQ1M,
$\sqrt{S}$=1.8 TeV
μ=M$_{jj}$/2

$(\sigma_{ij}^{(res)} - \sigma_{ij}^{(3)})/\sigma^{(3)}$

M$_{jj}$

Figure 6: *Contribution of gluon resummation at order α_s^4 and higher, relative to the truncated $\mathcal{O}(\alpha_s^3)$ result, for the invariant mass distribution of jet pairs at the Tevatron.*

12

References

[1] Yu.L. Dokshitzer, D.I. Dyakonov and S.I. Troyan, *Phys. Rep.* **58**(1980)271;
G. Parisi and R. Petronzio, *Nucl. Phys.* **B154**(1979)427;
G. Parisi, *Phys. Lett.* **B90**(1980)295;
G. Curci and M. Greco, *Phys. Lett.* **B92**(1980)175 and **B102**(1981)280;
D. Amati et al., *Nucl. Phys.* **B173**(1980)429;
M. Ciafaloni and G. Curci, *Phys. Lett.* **B102**(1981)352;
P. Chiappetta, T. Grandou, M. Le Bellac and J.L. Meunier, *Nucl. Phys.* **B207**(1982)251.

[2] G. Altarelli, R.K. Ellis and G. Martinelli, *Nucl. Phys.* **B157**(1979)461;
J. Kubar-André and F.E. Page, *Phys. Rev.* **D19**(1979)221.

[3] R. Hamberg, W.L. van Neerven, and T. Matsuura, *Nucl. Phys.* **B359**(1991)343;
W.L. van Neerven and E.B. Zijlstra, *Nucl. Phys.* **B382**(1992)11.

[4] G. Sterman, *Nucl. Phys.* **B281**(1987)310.

[5] S. Catani and L. Trentadue, *Nucl. Phys.* **B327**(1989)323 and **B353**(1991)183.

[6] E. Laenen, J. Smith and W.L. van Neerven, *Nucl. Phys.* **B369**(1992)543; *Phys. Lett.* **B321**(1994)254.

[7] E. Berger and H. Contopanagos, *Phys. Lett.* **B361**(1995)115; ANL-HEP-PR-95-82, hep-ph/9603326.

[8] G. Sterman, talk presented at the 10$^{\text{th}}$ Rencontres de Physique de la Vallée d'Aoste, La Thuile, Val d'Aosta, March 3–9, 1996;
N. Kidonakis and G. Sterman, ITP-SB-97-7, hep-ph/9604234;
H. Contopanagos, E. Laenen and G. Sterman, ANL-HEP-25, hep-ph/9604313.

[9] S. Catani, M. Mangano, P. Nason and L. Trentadue, preprint CERN-TH/96-86, hep-ph/9604351.

[10] S. Catani, M. Mangano, P. Nason and L. Trentadue, *Phys. Lett.* **B351**(1995)555.

[11] P. Nason, S. Dawson and R. K. Ellis, *Nucl. Phys.* **B303**(1988)607.

[12] D. Appel, G. Sterman and P. Mackenzie, *Nucl. Phys.* **B309**(1988)259.

[13] H. Contopanagos and G. Sterman, *Nucl. Phys.* **B419**(1994)77;
L. Alvero and H. Contopanagos, *Nucl. Phys.* **B436**(1995)184.

[14] M. Beneke and V.M. Braun, *Nucl. Phys.* **B454**(1995)253, hep-ph/9506452.

[15] G.P. Korchemsky and G. Sterman, *Nucl. Phys.* **B437**(1995)415.

[16] Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, preprint CERN-TH-95-281, hep-ph/9512336.

[17] R. Akhoury and V.I. Zakharov, *Phys. Lett.* **B357**(1995)646.

[18] R. Akhoury and V.I. Zakharov, UM-TH-95-33, Dec 1995, hep-ph/9512433

[19] R.K. Ellis and D.A. Ross, *Nucl. Phys.* **B345**(1990)79;
S. Catani, M. Ciafaloni and F. Hautmann, *Phys. Lett.* **B242**(1990)97, *Nucl. Phys.* **B366**(1991)135, *Nucl. Phys. (Proc. Suppl.)* **23A**(1991)328;
J.C. Collins and R.K. Ellis, *Nucl. Phys.* **B360**(1991)3.

[20] A.D. Martin, R.G. Roberts and W.J. Stirling, *Phys. Rev.* **D50**(1994)6734.

[21] N. Kidonakis and J. Smith, ITP-SB-95-16, hep-ph/9506253.

[22] F. Abe et al., CDF Coll., FNAL-Pub-96/020-E.

[23] F. Aversa, P. Chiappetta, M. Greco and J.P.Guillet, *Nucl. Phys.* **B327**(1989)105, *Phys. Rev. Lett.* **65**(1990)401;
S. Ellis, Z. Kunszt and D. Soper, *Phys. Rev.* **D40**(1989)2188, *Phys. Rev. Lett.* **64**(1990)2121.

[24] E. Buckley-Geer, for the CDF Collaboration, FERMILAB-CONF-95-316-E, Sep 1995, presented at International Europhysics Conference on High Energy Physics (HEP 95), Brussels, Belgium, 1995.

[25] S. Ellis, Z. Kunszt and D. Soper, *Phys. Rev. Lett.* **69**(1992)1496;
W.T. Giele, E.W.N. Glover and D.A. Kosower, *Phys. Rev. Lett.* **73**(1994)2019.

[26] J. Huston et al., MSU-HEP-50812, hep-ph/9511386;
H.L. Lai et al., MSU-HEP-60426, hep-ph/9606399.